In all areas of life, math helps people solve problems and make good decisions. In recognition of the need for math knowledge, the 1997 Amendments to the Individuals with Disabilities Education Act (IDEA) raised the bar on what students with disabilities are expected to learn. If students with disabilities are to achieve to their potential, they must have mathematical knowledge and skills as reflected in the general education curriculum.

A significant element of the standards-driven reform effort is the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (first published in 1989 and revised in 2000). The NCTM Standards focus on conceptual understanding and problem solving rather than procedural knowledge or rule-driven computation. Most states and districts have used the NCTM Standards to some degree in revamping their mathematics curricula. [For more information on NCTM Standards, visit the NCTM web site at http://standards.nctm.org.]

According to OSEP-funded researchers Paula Maccini and Joe Gagnon, most special education teachers do not have sufficient knowledge about the NCTM Standards and this undermines their ability to provide support to students with disabilities. “In our survey, we found that many special education teachers were unfamiliar with the NCTM Standards, and those who were felt they had insufficient information, support, and materials for implementing the standards,” Maccini reports. On the positive side, however, Maccini and Gagnon found a link between teacher familiarity with the NCTM Standards, teacher confidence teaching math, and student response. “Teachers who implemented activities, lessons, and strategies consistent with the NCTM Standards indicated that students with disabilities responded favorably,” Maccini says. “Many students with mild disabilities experience difficulties with the math curriculum, but with the right support, they can succeed in a higher level math curriculum,” Gagnon adds.

The challenge for teachers is to provide effective math instruction to students with disabilities so that they can meet high standards. Researchers featured in this Research Connections are helping us understand what it will take to ensure that students with disabilities succeed.
Knowing and Doing Math Improve Mathematics Achievement

With OSEP support, researchers are developing approaches that enhance students’ math performance by focusing on both the knowing and doing aspects of mathematics. The following examples show that effective math instruction must address the students’ deeper understanding of mathematics.

ENHANCING THE QUALITY OF MATHEMATICS FOR STUDENTS WITH DISABILITIES

“If we want students with disabilities to do better in math, teachers have to make it more accessible and meaningful to them,” asserts John Cawley, Professor Emeritus at the University of Connecticut. Cawley began his career in mathematics in 1953 and since that time has focused many of his academic pursuits on helping students with disabilities achieve mathematically. For him, both knowing and doing mathematics must be emphasized if we are to enhance the quality of mathematics instruction and learning for students with disabilities.

“Knowing about mathematics means the student comprehends the basic principles of the mathematics and knows there is more than one way to explain the mathematics and that there is frequently more than one acceptable answer,” Cawley explains. “This is contrasted with doing mathematics, which means the student can apply a number of different strategies and mathematics principles to complete an item. Today, many of the problems students face with math stem from educators neglecting the knowing and overemphasizing the doing.”

With OSEP support, Cawley has conducted many investigations that underscore the importance of helping students with disabilities know mathematics. Cawley shares an example that highlights the distinction between knowing about subtraction and being able to do subtraction. “Subtraction as a mathematical topic is much more meaningful than the rote computation, take away approach that has been advocated for students with disabilities since the 1920s,” Cawley points out. “The take away view of subtraction is limited and fails to assist students in achieving higher standards of mathematics knowledge and competence.” Cawley maintains that subtraction is a process that allows you to understand and find the difference between two numbers.

“The big idea for students to understand is that subtraction represents a difference,” Cawley states. “Knowing about subtraction involves reasoning in the form of proof and explanation. It also involves the ability to demonstrate the connectedness between one facet of mathematics (e.g., subtraction) and another (e.g., addition).”

Cawley has found that understanding subtraction in this way offers teachers numerous opportunities to stress number sense and skill development. “In developing an understanding of subtraction as a difference, students might analyze and discuss situations, such as determining what number must be added to another to make them the same, how much larger or smaller one number is in comparison to another, or what remains of a number after part of it has been taken away,” Cawley describes. “Teachers also may explore the reason there is a difference between two numbers; namely, that the sets of numbers
Students should be able to:
• Determine the differences between numbers.
• Use facts accurately and rapidly.
• Use multiple algorithms when subtraction involves two or more digits.
• Represent subtraction of two or more digits with expanded notation and use expanded notation to explain the transition from manipulative to traditional symbolic representations.
• Demonstrate knowledge that subtraction is not cumulative and that changing the order of numbers will change the answer.

This way of thinking also leads teachers to consider different approaches to subtraction, including troubleshooting problems students may be having. “Students with disabilities often have difficulty borrowing and renaming when doing subtraction,” Cawley says. “If we address the mathematics of this difficulty, we see that we can teach students to subtract without having to borrow.”

To understand Woodward’s approach, consider the case of Tina, a sixth grade student with mild disabilities who doesn’t know her multiplication tables. In Woodward’s approach, the easy multiplication facts (e.g., X1, X2) are identified as one distinct set. Tina passes the pretest, so she skips on to the harder facts. She also is shown strategies for learning facts, such as doubling. For example, Tina found it easier to learn 6X4 by starting with 6X2 and then doubling it. Another strategy Tina found useful involved helping or near facts. When Tina encountered a difficult fact, such as 6X7, she used a helping or near fact to solve it (e.g., Tina knew 6X6 = 36, so she added 6 to 6X7 to find the answer).

“Two related topics that are intimately related to a flexible understanding of numbers are the skills of mental computation and approximation,” Woodward says. Students are shown how they can apply their knowledge of basic facts to approximate the answers to problems with large numbers. For example, a problem such as 12,654 - 5,788 can be converted to approximate numbers. A beginning strategy might be to simply round up to 13,000 - 6,000 and use the fact 13 - 6 to yield an answer of 7,000. As students become more comfortable with mental calculations and approximations, other strategies for rounding numbers may be introduced (e.g., 654 and 788 are close in quantity and thus, it may be just as sensible to round to 12,000 - 5,000). “Such strategies help students learn to see other ways of looking at numbers,” Woodward summarizes.

Woodward and his colleagues have created a variety of curriculum materials that exemplify his approach. You can find a selection of them on his website at http://olsoncs.wou.edu/transmath/.

Thinking About Math Differently—One Teacher’s Experience

For Cal Young Middle School (Oregon) special education teacher Linda Vie, getting her students to think about math required some personal changes. “To use Woodward’s approach, I had to let go of some things I thought were important—like teaching long division without calculators, which, by the way, I am very good at! I realized that the students needed to understand the concept of division, and that once they understood it, they could do it on the calculator.”

Vie has integrated discussion about numbers into all aspects of her mathematics teaching. “When I started with the students, they could do some basic rote math facts okay, but they did not understand numbers.” Vie now pairs the methodical teaching of math facts with discussions about the meaning of...
Knowing and Doing Math (continued)

numbers. “When we are doing addition, we talk about the meaning of the numbers,” Vie explains. “We also are careful to refer to numbers correctly.” She cites the following example: When presented with an addition fact, 9 + 3, you often will hear the statement, “Carry the 1.” Yet, it is not a “one” that is being carried but a ten. Vie notes the importance of breaking down numbers with students to help them understand what they are doing. “Thinking about what the numbers mean lays the foundation for other skills—such as looking at an answer and asking, Does my answer make sense?”

Vie has carried these discussions into students’ work with math word problems. “Before we start solving the problem, we think about what the problem is asking. We look for necessary information, and sometimes even rewrite what the problem is asking in our own words,” Vie describes. “We then go through every sentence, asking if the information is necessary to solving the problem. Before students ever attempt to work the problem, they must know what is needed to work the problem.”

Mathematical Problem Solving Instruction

Tiki and Dale are going to the movies. They have $12 between them. Tickets cost $4.50 each. How much will they have left over for snacks?

Ms. Anderson needs to purchase enough paint to cover the walls and ceiling in an 8’ by 8’ room. One gallon of paint covers 200 square feet. How much paint does she need?

The Bailey family is planning a vacation to the city. The map shows the mileage to be 180 miles. If the family drives 55 miles per hour, 75 percent of the way and 35 miles per hour the rest of the distance, how much time will it take to get there?

Each day, we are presented with problems which require math. How can we help students to be better math problem solvers?

“We can start by understanding that students who have problems with mathematics typically do not know how to decide what to do,” points out University of Miami researcher Marjorie Montague. “Teaching students how to decide what to do is a cognitive process critical to instruction in mathematical problem solving.”

More than a decade ago, Montague received OSEP-funding to begin studying the problems students with disabilities have solving math word problems. Since then, her research has shown that effective and efficient mathematical problem solving depends on the ability to select and apply task-appropriate cognitive and metacognitive processes and strategies for understanding, representing, and solving problems. Montague describes cognitive processes as the “to do” strategies (see sidebar, Cognitive Processes Associated with Problem Solving), and metacognitive processes as the reflective strategies (e.g., “What am I doing?” and “What have I done?”).

To be good problem solvers, teachers need to know what good problem solvers do,” Montague says. To help teachers understand the knowledge and skills needed to be effective and efficient mathematical problem solvers, Montague developed the Solve It! approach. Solve It! is a research-based instructional program in which teachers explicitly teach the processes and strategies that underlie mathematical problem solving. It incorporates the cognitive processes critical to mathematical problem solving in each step of the strategy:

• Reading the problem. Students are taught how to read mathematical problems, including using reading strategies to understand the problem (e.g., focusing on important information), developing mathematical vocabulary, and recognizing when they do not understand relationships among mathematical terms.
and quantitative concepts expressed in a problem.

- **Paraphrasing.** Students are taught how to put the problem into their own words and convey meaning.

- **Visualizing.** Students are taught to draw a representation or to make a mental image of the problem.

- **Hypothesizing about problem solutions.** Students are taught how to decide the number of operations that are needed to solve the problem, select and order the operations, and then to transform the information into correct equations and algorithms.

- **Estimating the answer.** Students are taught how to stay focused on the type of outcome (e.g., number of yards rather than feet), and then how to predict the answer by using the information in the problem and their projected solution path.

- **Computing.** Students are taught how to recall the correct procedures for working through the algorithms and the necessary math facts for accuracy.

- **Checking the problem.** Students are taught how to check the mathematical problem solving process to ensure that they have understood the problem, accurately represented the problem, selected an appropriate solution path, and solved the problem correctly.

In the Solve It! approach, students also learn a metacognitive strategy that they apply at each step. The strategy includes the following steps:

- Say aloud or to themselves what the problem is asking them to do.
- Ask themselves if they understand the problem.
- Check their progress.

“Metacognitive strategies help problem solvers gain access to strategic knowledge, provide guidance in applying the cognitive strategy, and regulate the use of the cognitive strategies and overall performance,” Montague adds.

### A Look at Solve It! in the Classroom

According to Gretchen Daniel, it was easy getting her Dublin City (Ohio) Middle School students with learning disabilities hooked on Solve It! “When you hear sixth grade boys call math fun, you know something is working right!”

Daniel has been implementing the Solve It! approach in pull-out settings with students who receive resource room support for math difficulties. “I use explicit instruction to teach the strategy—I model each cognitive process, I have the students verbalize the cognitive activities until they become automatic, I provide opportunities for students to practice with their peers, and I continually monitor their progress and provide feedback.”

Although Daniel stresses the importance of replicating Montague’s approach as it was researched—“if you want the program to work, you need to implement it as it was written”—she added a few minor strategies to meet the individual needs of her students. “My sixth graders wanted to create a mnemonic for the cognitive steps (read, paraphrase, visualize, hypothesize, estimate, compute, check),” Daniel reports. “The students came up with *rhinos play vicious byenas, except competitive camels*—which they used until they internalized the steps.” In addition, Daniel addressed a motivational issue. “Many of my students had previously failed at math and needed help developing a can do attitude,” Daniel explained. The solution came in the form of affirmations that students internalized. “We made posters that stated Strategy Use = Success! Students also quickly adopted the phrase, I will use the strategy and do well.”

Overall, Daniel found that it took three to four sessions for students to master the strategy. “The key is to thoroughly understand the strategy yourself so that you can teach it and model it.”

### RESOURCE

**Reading and Writing the Braille Code of Mathematics**

Imagine you are unable to read or write the symbols that comprise mathematics. You would be forced to learn concepts and perform calculations entirely in your head, limiting your ability to master the intricacies of mathematics. Unfortunately, many students who are blind find themselves in this situation.

To address this need, OSEP has supported researchers Gaylen Kapperman and Jodi Sticken of Northern Illinois University in developing an interactive software tutorial that can be used by students who are blind. The software helps them to study the Nemeth Code (the Braille code for mathematics). The software is installed in a Braille Lite—a small, portable Braille notetaker that is equipped with synthetic speech and a refreshable Braille display. The speech and tactile Braille features enable students to study the Nemeth Code independently or under the direction of a teacher. A tutorial on the Nemeth Code for sighted individuals also is available.

The software program may be downloaded from Freedom Scientific at [www.freedomscientific.com/fs_downloads/notenemeth.asp](http://www.freedomscientific.com/fs_downloads/notenemeth.asp). The tutorial for sighted individuals is available from the Texas School for the Blind and Visually Impaired web site at [www.tsbsvi.edu/math/math-resources.htm#Download](http://www.tsbsvi.edu/math/math-resources.htm#Download). For more information, contact Kapperman [gkapper@niu.edu](mailto:gkapper@niu.edu) and Sticken [jsticken@niu.edu](mailto:jsticken@niu.edu) at the Department of Teaching and Learning, Northern Illinois University, DeKalb, IL 60115.
Helping Students with Disabilities Participate in Statewide Math Assessments

The 1997 Reauthorization of the Individuals with Disabilities Education Act (IDEA) provides that students with disabilities will participate in state and district-wide assessments, with accommodations and modifications as necessary. These large-scale assessments reflect standards that all students are expected to meet. Most State and district-wide assessments tap mathematical knowledge and skills. Emerging research is shedding light on what practitioners can do to ensure that students with disabilities participate and achieve their potential on such assessments.

MAKING ASSESSMENT ACCOMMODATIONS FOR MATH ASSESSMENTS

"IDEA '97 has heightened the need for research findings on the effects of assessment accommodations," says Martha Thurlow, director of the National Center on Educational Outcomes (NCEO). "The complexity of the many studies on test changes made it evident that a searchable data base was needed to cull the information for addressing specific accommodations, specific groups of students, specific ages, or combinations of these and other factors." Under Thurlow's leadership, NCEO launched a searchable data base of accommodations (http://education.umn.edu/NCEO/AccomStudies.htm).

The online data base houses an accommodations bibliography that allows users to search a compilation of empirical research studies on the effects of various testing accommodations for students with disabilities.

"You can search the bibliography for specific accommodation research studies by typing in keywords related to the accommodation, disability, test content area, or student age," Thurlow describes. Currently, the data base contains 173 documents, covering the years through 2001.

STUDENT PERFORMANCE ON MATH ASSESSMENT: IMPLICATIONS FOR STANDARDS

"If students with disabilities are to meet high standards, they will need support," says Rene Parmar.

With OSEP funding, Parmar and her colleagues Barbara Signer and John Cawley set out to explore the discrepancy between the desire for higher standards of student performance in grade levels, the difference between the scores of students with disabilities and their peers was even greater.

• On state benchmarks for mathematics proficiency, the percentage of all students meeting criteria for proficiency ranged from 11% to 87%, whereas the percentage of students with disabilities meeting proficiency requirements ranged from 2% to 77%. Beyond elementary school, only a fraction of students with disabilities met proficiency in any state (with the exception of middle school students in Texas).
and the curriculum guides of the states.

Parmar sees other practical implications as emerging from this research. “Students need support in thinking about and understanding math concepts,” According to Parmar, traditional assessments that yield pass-fail data are rarely useful instructionally. “Even with rubrics, teachers seldom have sufficient information to identify specific student difficulties that require instructional intervention,” Parmar asserts. “Teachers need information that answers questions such as, What is the student thinking when he or she is encountering math?”

One suggestion for teachers is to assess the student by testing with items that occur between the last item correct and the first item failed to determine the type of item and type of error. Using the principle of least error correction, the teacher determines if the error was one of calculation or faulty use of an algorithm, and then corrects only the dominate error. For example, if a student completes an item by going from left-to-right and makes an error in calculation, use of the left-to-right algorithm would not be addressed.

When assessing student errors, Parmar encourages teachers to talk to students about what they are thinking. “Teachers need to understand the cognitive aspects of the math problem so that they can intervene if necessary.” For example, when solving story problems, consider the following:

- **Problem definition.** If the student knows that addressing the problem requires math, does he or she know how to apply math (e.g., multiply, calculate an angle)? Some students do not know or they guess. They continue to fail because they started out thinking erroneously.

- **Problem comprehension.** Does the student comprehend the language of math? Phrases may have special meaning in math, such as “of these,” which means a subset in math.

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**RESOURCE**

**Math Fluency**

According to researcher Ted Hasselbring of the University of Kentucky, students with disabilities often have difficulty developing fluency. Students without math disabilities can recall more facts from memory than their peers with math disabilities. This discrepancy increases with age, resulting in students falling further and further behind in their ability to recall basic math facts from memory.

To help alleviate this, Hasselbring and his colleagues created the **Math Fluency Program** with OSEP funding. The program—which is still as current today as it was more than a decade ago—provides systematic instruction and practice for developing student ability to recall the answers to basic math facts accurately and fluently. It embodies several unique design features that make it particularly attractive to students with disabilities.

For more information, contact Hasselbring at Special Education Technology, University of Kentucky, 229 Taylor Education Building, Lexington, KY 40506, tsh@uky.edu.

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Mathematics and current data showing that students with disabilities tend not to meet expectations. “We looked at mathematical proficiency in relation to NCTM standards, which are aligned into groupings for PreK-2, 3-5, 6-8, and 9-12,” Parmar explains. “We found that significant numbers of general education students and those with mild disabilities do not demonstrate proficiency in many of the topics introduced or expected to be mastered (e.g., division) at a specific grade level.” For Parmar, the implication is that schools should replace the grade-by-grade level system with a multigrade level format. “Unfortunately,” Parmar adds, “the commercial materials provided to the schools and the curriculum guides of the states and districts continue to specify grade-by-grade level content.”

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**RESOURCE**

**Meet the Math Wiz**

Using multimedia capability, OSEP-funded researchers Jean Andrews and Donald Jordan at Lamar University, developed the **Meet the Math Wiz** CD-ROM series for students who use American Sign Language (ASL) to communicate. In addition to signing the content, the program also is translated in Spanish.

**Meet the Math Wiz** helps students focus on math word problems over six grades of math difficulty using multicultural names, stories, and themes. For example, the program features Chris Kurtz, a math teacher who is deaf. He welcomes users to his castle, where he describes, among other things, a four-point plan for solving math word problems. He leads users into eight demonstrations per CD, giving them an ASL translation of the problem, an animation hint, and an explanation of how to solve the problem in ASL. Math words are linked to an ASL sign and explanation dictionary.

Materials can be ordered from: Curriculum Publications Clearinghouse, Horrabin Hall 46, Western Illinois University, 1 University Circle, Macomb, IL 61455, 800-322-3905. For more information, contact Andrews at jandrews47@aol.com.
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Marjorie Montague served as a guest consultant to this issue.